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New Laser Properties and Spectroscopy of Orthorhombic Crystals $YAlO_3:Er^{3+}$

Intensity Luminescence Characteristics, Stimulated Emission, and Full Set of Squared Reduced-Matrix Elements $|\langle\alpha[SL]\,J|\,|U^{(t)}|\,|\alpha'[S'L']\,J'\rangle|^2$ for Er^{3+} Ions

BY

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New laser data on orthorhombic $YAlO_3:Er^{3+}$ crystals are obtained. Stimulated emission in the ${}^4S_{3/2} \to {}^5I_{15/2}$ channel and cascase lasing of the sequential intermanifold ${}^4S_{3/2} \to {}^4I_{11/2} \to {}^4I_{13/2}$ transitions are excited at ≈ 110 K with Xe-flashlamp pumping. Intensity absorption and luminescence characteristics of Er^{3+} ions in the $YAlO_3$ crystal are experimentally determined and quantitatively analyzed in terms of the known semiempirical method. The intensity spectroscopic parameters Ω_t obtained ($\Omega_2 = 0.95$, $\Omega_4 = 0.58$, and $\Omega_6 = 0.55$ (in 10^{-20} cm^2)) nicely describe band-area intensities in the absorption spectrum of the $YAlO_3:Er^{3+}$ crystal in the spectral region below 30000 cm^{-1}. A full set of reduced-matrix elements for the Er^{3+} ions is calculated involving all 41 J-manifolds of the $4f^{11}$ configuration lying in energy up to 97000 cm^{-1}. Using these data, the earlier reported intensity parameter Ω_t for the $YAlO_3:Er^{3+}$ crystal are revised and it is shown that involving highly excited levels of Er^{3+} ions into intensity spectroscopic analysis leads to an overestimation of the parameters Ω_t because of the possible presence of some additional absorption sources in the $YAlO_3$ host.

1. Introduction

Of the oxide compounds activated with trivalent lanthanide ions (Ln^{3+}) most widely used in quantum electronics, $RAlO_3$ orthorhombic crystals, where R = Y and Ln, ($D_{2h}^{16} - P_{bnm}$ space group) exhibit the largest number (at present 21) of intermanifold generating channels. Stimulated emission (SE) for most of them can be excited at room temperature using lamp pumping [1, 2]. These crystals have a less extended phonon spectrum as compared with that of garnet crystals, another family of popular oxide laser compounds, that makes them more attractive for SE excitation of Ln^{3+} ions in various spectral ranges including the visible and mid-IR. In this family of laser materials, $YAlO_3$ crystals are most prominent because they have a very favorable combination of high mechanical hardness, considerable heat conductivity, and optical properties.

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Table 1
Well-known lasing orthorhombic aluminates doped with Ln^{3+} ions

crystal host	Ln^{3+} activator ion				
	Pr^{3+}	Nd^{3+}	Ho^{3+}	Er^{3+}	Tm^{3+}
$YAlO_3$	+	+	+	+	+
$(Y, Er)AlO_3$			+	+	+
$GdAlO_3$		+	+	+	+
$ErAlO_3$			+	+	+
$(Er, Lu)AlO_3$			+		+
$LuAlO_3$	+	+	+	+	+

Lasing properties of these promising crystals have been discovered in [3] where SE of Nd^{3+} ions was excited for the first time ($^4F_{3/2} \rightarrow {}^4I_{11/2}$ channel at 300 K). In the chronological sequence, Er^{3+} was the second Ln^{3+} ion generating in $YAlO_3$ crystals ($^4S_{3/2} \rightarrow {}^4I_{9/2}$) [4] and the next ions were Tm^{3+} ($^3F_4 \rightarrow {}^3H_5$) [5], Ho^{3+} ($^5I_6 \rightarrow {}^5I_7$) [6], and Pr^{3+} ($^3P_0 \rightarrow {}^3F_3$ and $^3P_0 \rightarrow {}^3F_4$) [7]. Orthorhombic aluminates doped with Ln^{3+} activators are also able to generate on the cascade and cross-cascade operating schemes, as wells with laser-diode pumping and at other experimental conditions (see, for instance, [8 to 12] and Table 1). However, of the $YAlO_3 : Ln^{3+}$ system, only the $YAlO_3 : Nd^{3+}$ crystals appear to be well studied spectroscopically (a full list of main references is presented in [1, 13]). This paper was stipulated by increasing attention of experts and our own interest to laser potentialities of the $YAlO_3 : Er^{3+}$ crystals.

In the preceding publications, some experimental spectroscopic data for Er^{3+} ions in $YAlO_3$ crystals were obtained (see, for example, [14 to 23]) which reveal the largest number of generating intermanifold $J \rightarrow J'$ transitions among known oxide compounds (six channels are presently known). Unfortunately, this information is not enough to study in detail the absorption and luminescence intensity characteristics of this extremely interesting laser activator and, especially, its emitted intermanifold transitions involving high-lying states. Another important complicating problem is the lack of systematized data on the reduced-matrix elements of the unit tensor operator for high-lying manifolds (with $E_J > 30000\ \mathrm{cm}^{-1}$). These data are urgently needed for theoretical estimations of intensity characteristics of luminescence transitions by the well-known method [1, 26 to 28] based on the approach [24, 25]. That is why in this paper, in addition to our new data on SE generation of Er^{3+} ions ($^4S_{3/2} \rightarrow {}^4I_{15/2}$ and $^4S_{3/2} \rightarrow {}^4I_{11/2} \rightarrow {}^4I_{13/2}$ channels) in $YAlO_3$ crystals and results of intensity analysis of absorption and luminescence intermanifold transitions, we have presented a full set of squared reduced-matrix elements $|\langle 4f^{11}\alpha[SL]\,J|\,|U^{(t)}|\,|4f^{11}\alpha'[S'L']\,J'\rangle|^2$ involving all 41 J-manifolds of the $4f^{11}$ configuration of the Er^{3+} ions lying in energy up to $97000\ \mathrm{cm}^{-1}$. These data are of fundamental importance because they provide a useful basis for the further theoretical treatment of optical intensity characteristics of Er^{3+} ions in crystals related to its highly excited states involved, in particular, in upconversion processes. In this theoretical part of the paper, we continue our systematical calculations of full sets of reduced-matrix elements $\langle \| U^{(t)} \| \rangle$ for Ln^{3+} ions for the whole lanthanide series that have been started in our previous paper for Nd^{3+} ions [29].

2. Laser and Spectroscopic Measurements

To carry out low-temperature generating experiments, active elements shaped as a rod of 40 mm in length and 5 mm in diameter were fabricated from $YAlO_3$ single crystals ($C_{Er} = 0.5$ to 1.5 at%) having the laser axis parallel to the [112] crystallographic direction. In these measurements, two essential problems were solved, the first one concerns the generation of pulse SE in the green spectral region in inter-Stark transitions of the $^4S_{3/2} \rightarrow {}^4I_{15/2}$ channel, and the second one is to obtain generation in the direct cascade scheme $^4S_{3/2} \rightarrow {}^4I_{11/2} \rightarrow {}^4I_{13/2}$. To accomplish these ends we used a highly efficient, elliptical cross-section illuminating chamber and a pulse Xe-flashlamp (ISP-250 type with $\tau_{exc} \approx 70$ μs) and a glass tube cryostat [30]. The active element in the latter was cooled (to ≈ 110 K) by a flow of liquid nitrogen vapor. A confocal optical resonator was formed by changeable spherical mirrors ($R = 500$ mm) with an interference dielectric coating having a transmission of about 0.5% at the SE wavelengths. Spectral composition and kinetics of SE generation were measured using a grating MDR-3 monochromator and a cooled InSb photoresistor equipped with the corresponding electronics. A crystal having the concentration of the activator $C_{Er} \approx 0.5$ at% was used to generate the SE of Er^{3+} ions on inter-Stark transitions of the resonance $^4S_{3/2} \leftrightarrow {}^4I_{15/2}$ channel, whereas in the cascade laser experiments the concentration was $C_{Er} \approx 1.5$ at%. All the crystals were subjected to a special annealing to prevent formation of undesirable color centers arising under the influence of the short-wavelength spectrum of the pumping Xe-flashlamp. In this stage of the study, the plane-parallel ends ($\approx 10''$) of the active elements have no anti-reflection coating. The main results of laser measurements are listed in Table 2. Note that SE of Er^{3+} ions in the $YAlO_3$ crystals at the intermanifold $^4S_{3/2} \rightarrow {}^4I_{15/2}$ transition was earlier excited at 77 K in the upconversion scheme using laser pumping [9], and individual generation of the $^4S_{3/2} \rightarrow {}^4I_{11/2}$ and $^4I_{11/2} \rightarrow {}^4I_{13/2}$ channels was obtained in [31].

In the theoretical analysis of spectroscopic intensity characteristics of Er^{3+} ions in the orthorhombic aluminate crystal, we used a set of oscillator strenghts $f_{JJ'}^{exp}$ measured by us, averaged over three crystallographic axes a, b, and c. In accordance with [32, 33] these values were determined in [1, 26 to 28] with the experimental absorption $^4I_{15/2} \rightarrow J'$ band areas of the corresponding intermanifold transitions (see column 4, Table 3) measured on a grating spectrophotometer (model Cary-2300) using oriented plane-parallel $YAlO_3:Er^{3+}$ plates ($C_{Er} \cong 1.5$ at%) and averaged refractive indices $\bar{n}$ (column 3, Table 3) based on data of [34].

Table 2

Some characteristics of pulse laser performance of Er^{3+} ions in orthorhombic crystals $YAlO_3$ at ≈ 110 K

C_{Er} (at%)	SE channel	λ_{SE}[a] (μm)	E_{thr}[b] (J)	E_{term}[c] (cm^{-1})
≈ 0.5	$^4S_{3/2} \rightarrow {}^4I_{15/2}$	0.5500	≈ 65	≈ 218
≈ 1.5	$^4S_{3/2} \rightarrow {}^4I_{11/2}$	1.2390[d]	≈ 15	≈ 10330
	$^4I_{11/2} \rightarrow {}^4I_{13/2}$	2.7398 [d]	≈ 15	$\approx$ 6637

[a] λ_{SE} is the SE wavelength in the free-running pulse mode, accuracy of measurements is ±0.0003 μm.

[b] E_{thr} is the threshold energy of SE excitation.

[c] E_{term} is the energy of terminal Stark laser level.

[d] SE was excited on direct cascade laser scheme.

Table 3

Calculated intensity parameters Ω_t and absorption intensity characteristics of $^4I_{15/2} \rightarrow J'$ channels (band areas) of Er^{3+} ions in orthorhombic $YAlO_3$ crystals at 300 K

J' manifold	$\bar{\lambda}$ (μm)	$\bar{n}$	data of our measurements			results of checking of the data from [27]					
			$10^7 \bar{f}^{exp}_{JJ'}$	$s^{ed}_{JJ'}$ $(10^{-20}\ cm^2)$		$10^7 \bar{f}^{exp}_{JJ'}$	$s^{ed}_{JJ'}$ $(10^{-20}\ cm^2)$			$10^7 \bar{f}^{ed}_{JJ'}$ (calc)	
				exp.	calc.		exp.	calc.*)	calc.**)	data of [27]	our data
1	2	3	4	5	6	7	8	9	10	11	12
$^4I_{13/2}$	1.55	1.920	–	–	–	12.7	1.526	1.230	–	6.3^{ed}	12.3
$^4I_{11/2}$	0.99	1.926	3.3	0.248	0.243	3.1	0.236	0.311	0.248	2.3	4.5
$^4I_{9/2}$	0.87	1.928	1.7	0.102	0.105	2.5	0.153	0.228	0.126	4.3	7.4
$^4F_{9/2}$	0.66	1.937	12.3	0.525	0.561	11.5	0.588	1.027	0.632	17.5	34.8
$^4S_{3/2}$	0.55	1.946	3.3	0.142	0.121	4.1	0.175	0.165	0.124	2.1	4.1
$^2H(2)_{11/12}$	0.53	1.948	23.9	0.971	0.966	24.3	0.989	0.992	0.990	24.1	47.2
$^4F_{7/2}$	0.49	1.953	10.2	0.387	0.427	11.2	0.425	0.655	0.454	12.0	23.1
$^4F_{5/2} + {}^4F_{3/2}$	0.46	1.960	5.9	0.206	0.192	6.0	0.209	0.262	0.197	4.1	7.8
$^2H(2)_{9/2}$	0.41	1.970	3.9	0.123	0.134	4.2	0.134	0.193	0.140	3.8	7.1
$^4G_{11/2}$	0.38	1.979	41.9	1.233	1.240	43.1	1.270	1.269	1.271	43.4	83.4
$^4G_{9/2} + {}^2K_{15/2} + {}^2G(1)_{7/2}$	0.36	1.986	15.9	0.449	0.345	16.7	0.473	0.583	0.380	17.7	34.5
$^2P_{3/2}$	0.32	2.009	–	–	–	0.43	0.010	0.0001	–	0.29	0.5
$^4G_{7/2}$	0.30	2.026	–	–	–	14.6	0.334	0.045	–	20.6	4.0
$^2D(1)_{5/2}$	0.29	2.031	–	–	–	0.58	0.013	0.017	–	0.41	0.80
$^2G(1)_{9/2}$	0.27	2.046	–	–	–	3.0	0.064	0.064	–	3.2	6.2
$^4D_{5/2} + {}^4D_{7/2}$	0.26	2.066	73.3	–	–	73.3	1.478	1.178	–	62.7	36.2
Intensity parameters $(10^{-20}\ cm^2)$											
Ω_2					0.95			0.56	0.91		
Ω_4					0.58			1.27	0.70		
Ω_6					0.55			0.75	0.56		
rms deviation $(10^{-20}\ cm^2)$					0.046			0.20	0.046		

*) Calculated with use of 16 absorption $^4I_{15/2} \rightarrow J'$ band areas. The intensity parameters from [27] are (in $10^{-20}\ cm^2$): $\Omega_2 = 1.06$, $\Omega_4 = 2.36$, and $\Omega_6 = 0.78$.

**) Calculated with use of ten absorption $^4I_{15/2} \rightarrow J'$ band areas.

3. Analysis of Intensity Absorption and Luminescence Characteristics of the $YAlO_3:Er^{3+}$ Crystal

Intensity spectroscopic characteristics of the $YAlO_3:Er^{3+}$ crystal were, as mentioned above, analyzed by the method based on the theoretical approach [24, 25]. Because the detailed description of the corresponding procedure is available elsewhere (see, e.g., [1, 26 to 28]), only a brief outline of its background is given here. In this method, the electric-dipole (ed) line strengths $s^{\mathrm{ed}}_{JJ'}$ of the intermanifold transition between the initial $|4f^N\alpha[SL]\,J\rangle$ and final $|4f^N\alpha'[S'L']\,J'\rangle$ states (where S, L, and J are the total spin, the total orbital momentum, and the total angular momentum, respectively, whereas α is an additional index, classifying Russel-Saunders (RS) manifolds for repeated RS terms of the $4f^N$ electronic configuration with the same quantum numbers S and L) of a Ln^{3+} ion is defined by

$$s^{\mathrm{ed}}_{JJ'} = \sum_{t=2,4,6} \Omega_t\, |\langle 4f^N\alpha[SL]\,J|\,|U^{(t)}|\,|4f^N\alpha'[S'L']\,J'\rangle|^2\,, \tag{1}$$

where Ω_t are intensity parameters and $\langle\|U^{(t)}\|\rangle$ are reduced-matrix elements of the unit tensor operator $U^{(t)}$ of rank t.

The semi-phenomenological intensity parameters Ω_t were calculated by a least-square fit between the theoretical line strengths (1) and those derived from the experimental oscillator strengths $\bar{f}^{\mathrm{exp}}_{JJ'}$ using the known relation

$$\bar{f}^{\mathrm{ed}}_{JJ'} = \frac{8\pi^2 mc}{3h(2J+1)\,\bar{\lambda}}\left[\frac{(\bar{n}^2+2)^2}{9\bar{n}}\right] s^{\mathrm{ed}}_{JJ'}\,, \tag{2}$$

where $J = 15/2$ is the total angular momentum of the $^4I_{15/2}$ ground state of the activator Er^{3+} ions, $\bar{\lambda}$ and $\bar{n} = n(\bar{\lambda})$ are the mean wavelength of the absorption $^4I_{15/2} \to J'$ band areas and the refractive index of the crystal at wavelength $\bar{\lambda}$, respectively. In their turn, the oscillator strengths $\bar{f}^{\mathrm{ed}}_{JJ'}$ were obtained from the absorption spectra of $YAlO_3:Er^{3+}$ crystals using the formula

$$\bar{f}^{\mathrm{ed}}_{JJ'} = N_0^{-1}\,\frac{mc}{\pi e^2}\left[\frac{9\bar{n}}{(\bar{n}^2+2)^2}\right]\int k(\lambda)\,\mathrm{d}\lambda\,, \tag{3}$$

where N_0 is the number of Er^{3+} ions per cm^3 of the host crystal, and $\int k(\lambda)\,\mathrm{d}\lambda$ is the integrated absorption coefficient referred to the corresponding absorption $^4I_{15/2} \to J'$ band areas (column 1, Table 3). The $\int k(\lambda)\,\mathrm{d}\lambda$ values were calculated from the absorption spectra of $YAlO_3:Er^{3+}$ crystals using a standard graphical integration procedure whose accuracy is normally some 10%.

The total radiative probability $A_{JJ'}$ of intermanifold $J \to J'$ transitions for Ln^{3+} ions in crystals is the sum of ed and magnetic dipole (md) transition probabilities, $A^{\mathrm{ed}}_{JJ'}$ and $A^{\mathrm{md}}_{JJ'}$, respectively; it may be calculated using the equation

$$A_{JJ'} = A^{\mathrm{ed}}_{JJ'} + A^{\mathrm{md}}_{JJ'} = \frac{64\pi^4 e^2}{3h(2J+1)\,\bar{\lambda}^3}\left[\chi^{\mathrm{ed}} s^{\mathrm{ed}}_{JJ'} + \chi^{\mathrm{md}} s^{\mathrm{md}}_{JJ'}\right], \tag{4}$$

where $\chi^{\mathrm{ed}} = (\bar{n}^2+2)^2\,\bar{n}/9$ and $\chi^{\mathrm{md}} = \bar{n}^3$ are Lorentz-field correction factors for the refractivity of the medium for ed and md transitions, respectively, $s^{\mathrm{ed;md}}_{JJ'}$ is the line strength, and J the total angular momentum of the initial luminescence state involved in the intermanifold $J \to J'$ transition (all other notations in (4) have their usual meaning).

In this paper, a theoretical anlysis of all important intensity characteristics of the $YAlO_3:Er^{3+}$ crystals was carried out using a new full set of $\langle\|U^{(t)}\|\rangle$ matrix elements for

Er^{3+} ions calculated by us. Details of the corresponding calculations are discussed in the next section.

The line strength $s_{JJ'}^{\mathrm{md}}$ of the corresponding md intermanifold $J \to J'$ transition is defined by

$$s_{JJ'}^{\mathrm{md}} = \left(\frac{eh}{4\pi mc}\right)^2 |\langle 4f^N\alpha[SL]\,J|\,|\boldsymbol{L} + 2\boldsymbol{S}|\,|4f^N\alpha'[S'L']\,J'\rangle|^2\,, \tag{5}$$

where $\boldsymbol{L}$ and $\boldsymbol{S}$ are the operators of the total orbital momentum and total spin, respectively, and $\langle\|\boldsymbol{L} + 2\boldsymbol{S}\|\rangle$ is a reduced-matrix element of the operator $\boldsymbol{L} + 2\boldsymbol{S}$. These matrix elements were calculated using the eigenfunctions obtained from the diagonalization of the atomic Hamiltonian of the $4f^{11}$ configuration using the free-ion parameters for Er^{3+} ions in LaF_3 crystal [36].

For all known initial lasing states of Er^{3+} ions in crystals [1, 2] the intermanifold luminescence branching ratios $\beta_{JJ'}$ are calculated using the formula

$$\beta_{JJ'} = \frac{A_{JJ'}^{\mathrm{ed}} + A_{JJ'}^{\mathrm{md}}}{\sum\limits_{J'} (A_{JJ'}^{\mathrm{ed}} + A_{JJ'}^{\mathrm{md}})}\,, \tag{6}$$

and their radiative lifetime τ_{rad} of the initial luminescence J state is defined by

$$\tau_{\mathrm{rad}} = \frac{1}{\sum\limits_{J'} (A_{JJ'}^{\mathrm{ed}} + A_{JJ'}^{\mathrm{md}})}\,. \tag{7}$$

A least-square fit to the absorption data for the whole spectrum of $YAlO_3:Er^{3+}$ crystals obtained up to energies $\approx 40000\ \mathrm{cm}^{-1}$ and above yields a rather poor agreement between calculated $s_{JJ'}^{\mathrm{ed}}$ (calc) and measured $s_{JJ'}^{\mathrm{ed}}$ (exp) line strengths, and leads to a rather large root mean square (rms) deviation ($0.20 \times 10^{-20}\ \mathrm{cm}^2$). We have concluded from a careful analysis of the absorption spectra of $YAlO_3:Er^{3+}$ crystals that the reason for this may lie in the fact that, besides Er^{3+} activator ions, there are also some other sources of absorption in $YAlO_3$ crystals (the possible reasons may be the wing of the $4f^{10}5d$ absorption band [21], admixture ions, defects, etc.) whose extinction coefficients in the near UV become rather large and superimpose on the $4f^{11}$–$4f^{11}$ transition intensities. We suggest therefore that the data on absorption intensities for high-lying J manifolds of Er^{3+} ions in $YAlO_3$ cannot be regarded as quite reliable. Taking this into account, we omitted the high-lying manifolds from the consideration and calculated intensity parameters for the absorption spectrum cut at $\approx 30000\ \mathrm{cm}^{-1}$. This calculation resulted in a much better agreement betwen calculated and experimental line strengths (see columns 5 and 6, Table 3) as compared with those for the full absorption spectrum involving eleven band areas measured by us and, particularly, the rms deviation reduced by a factor of about four ($0.046 \times 10^{-20}\ \mathrm{cm}^2$). The corresponding intermanifold ed and md radiative transition probabilities $A_{JJ'}^{\mathrm{ed,md}}$ and luminescence branching ratios $\beta_{JJ'}$ as well as lifetimes $\tau_{\mathrm{rad}}^{\mathrm{calc}}$ are listed in Table 4.

It should be emphasized that our intensity parameters Ω_t (see column 6, Table 3): $\Omega_2 = 0.95$, $\Omega_4 = 0.58$, and $\Omega_6 = 0.55$ (in $10^{-20}\ \mathrm{cm}^2$) differ greatly from those reported in [27], ($\Omega_2 = 1.06$, $\Omega_4 = 2.36$, and $\Omega_6 = 0.78$ (all in $10^{-20}\ \mathrm{cm}^2$)). To examine the reason for this discrepancy, we tried to reproduce the data of [27] (see column 7, Table 3). However, having used the oscillator strengths reported in [27] and the corresponding matrix elements taken from [35]⁵), we obtained Ω_t parameters (column 9, Table 3): $\Omega_2 = 0.56$, $\Omega_4 = 1.27$,

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⁵) In fact, the restricted set of the reduced-matrix elements $\langle\|U^{(t)}\|\rangle$ reported in [35] for the ${}^4I_{15/2} \to J'$ transitions of the Er^{3+} ion virtually coincides with our full set of matrix elements which are reported in the next section, Table 5.

Table 4

Calculated intensity radiative characteristics, $A^{ed}_{JJ'}$ and $A^{md}_{JJ'}$, τ^{calc}_{rad}, and $\beta_{JJ'}$ of $J \rightarrow J'$ channels originating from the $^2P_{3/2}$, $^2H(2)_{9/2}$, $^4S_{3/2}$, $^4F_{9/2}$, $^4I_{9/2}$, $^4I_{11/2}$, and $^4I_{13/2}$ manifolds, as well as luminescence lifetime τ_{lum} and multiphonon nonradiative probabilities $W_{JJ'}$ for Er^{3+} ions in orthorhombic $YAlO_3$ crystals at 300 K

J	J'	$E_{JJ'}$	$A_{JJ'}$ (s^{-1})			τ^{calc}_{rad}	$\beta_{JJ'}$	τ^{exp}_{lum}	$W_{JJ'}$
		(cm^{-1})	ed	md	ed + md	(ms)	(%)	(ms)	(s^{-1})
$^4I_{13/2}$	$^4I_{15/2}$	6500	86.9	71.2	157.2	6.36	100	5.3 to 7.2 [16, 19, 23]	–
$^4I_{11/2}$	$^4I_{15/2}$	10100	105.6	–	135.3	7.39	78.1	0.9 to 1.2	$\approx 10^3$
	$^4I_{13/2}$	3600	14.3	15.4			21.9	[6, 16, 19]	
$^4I_{9/2}$	$^4I_{15/2}$	12250	98.9	–	139.1	7.19	71.1	$\approx$0.001	$\approx 6 \times 10^5$
	$^4I_{13/2}$	5750	38.6	–			27.7	[19]	
	$^4I_{11/2}$	2150	0.6	1.1			1.2		
$^4F_{9/2}$	$^4I_{15/2}$	15150	1009.7	–	1124.5	0.89	89.8	0.02	$\approx 7 \times 10^4$
	$^4I_{13/2}$	8650	44.5	–			4.0	[16]	
	$^4I_{11/2}$	5050	49.6	15.4			5.8		
	$^4I_{9/2}$	2850	1.7	3.6			0.4		
$^4S_{3/2}$	$^4I_{15/2}$	18350	981.2	–	1462.8	0.68	67.1	0.12 to 0.14	$\approx 8 \times 10^3$
	$^4I_{13/2}$	11850	398.6	–			27.2	[16, 19, 31]	
	$^4I_{11/2}$	8250	30.2	–			2.1		
	$^4I_{9/2}$	6100	52.3	–			3.6		
	$^4F_{9/2}$	3200	0.6	–			$\approx$0		
$^2H(2)_{9/2}$	$^4I_{15/2}$	24400	1024.3	–	2737.9	0.37	37.4	$\approx$0.001	$\approx 8 \times 10^5$
	$^4I_{13/2}$	17900	1013.0	–			37.0	[16]	
	$^4I_{11/2}$	14300	225.5	257.0			17.6		
	$^4I_{9/2}$	12150	18.0	1.4			0.7		
	$^4F_{9/2}$	9250	17.4	157.7			6.4		
	$^4S_{3/2}$	6050	0.3	–			$\approx$0		
	$^2H(2)_{11/2}$	5250	12.5	1.3			0.5		
	$^4F_{7/2}$	4100	4.0	4.3			0.3		
	$^4F_{5/2}$	2450	0.2	–			$\approx$0		
	$^4F_{3/2}$	2100	0.1	–			$\approx$0		
$^2P_{3/2}$	$^4I_{15/2}$	31550	387.5	–	4213.5	0.24	9.2	0.049	$\approx 5 \times 10^2$
	$^4I_{13/2}$	25050	1666.1	–			39.5	[16]	
	$^4I_{11/2}$	21450	1025.0	–			24.3		
	$^4I_{9/2}$	19250	271.1	–			6.4		
	$^4F_{9/2}$	16400	173.4	–			4.1		
	$^4S_{3/2}$	13200	226.7	39.2			6.3		
	$^2H(2)_{11/2}$	12400	58.2	–			1.4		
	$^4F_{7/2}$	11250	43.7	–			1.0		
	$^4F_{5/2}$	9600	45.9	65.9			2.7		
	$^4F_{3/2}$	9250	11.6	72.3			2.0		
	$^2H(2)_{9/2}$	7150	120.4	–			2.9		

and $\Omega_6 = 0.75$ (in 10^{-20} cm^2) that are quite different from those of [27]. Furthermore, we also recalculated the oscillator strengths $\bar{f}^{\text{ed}}_{JJ'}$ (calc) using the formula

$$\bar{f}^{\text{ed}}_{JJ'}\,(\text{calc}) = \frac{8\pi^2 mc}{3h(2J+1)\bar{\lambda}}\left[\frac{(\bar{n}^2+2)^2}{9\bar{n}}\right] \times \sum_{t=2,4,6} \Omega_t\,|\langle 4f^N\alpha[SL]\,J|\,|U^{(t)}|\,|4f^N\alpha'[S'L']\,J'\rangle|^2 \tag{8}$$

and the intensity parameters Ω_t of [27]. Again, our calculation (column 12, Table 3) did not reproduce the oscillator strengths of [27] (columns 11, Table 3). Because the least-square fit procedure for intensity spectroscopic characteristics always leads to an unambiguous result, we believe that there was a technical error in the intensity calculations of [27] (this error most likely takes its origin in the missing refractive index $\bar{n} \cong 1.94$ of the YAlO$_3$ crystal in the denominator of the Lorentz-field correction factor, $(\bar{n}^2 + 2)^2/9\bar{n}$, because for most of the $J \to J'$ transitions the ratio between the calculated oscillator strengths of [27] and our revised ones is close to this value).

On the other hand, the experimental $\bar{f}^{\text{exp}}_{JJ'}$ values of [27] are quite similar to our relevant data (compare columns 7 and 4, Table 3), so the corresponding intensity analysis for the spectrum of [27] truncated at $\approx$30000 cm^{-1} yields intensity parameters Ω_t: $\Omega_2 = 0.91$, $\Omega_4 = 0.70$, and $\Omega_6 = 0.56$ (in 10^{-20} cm^2) that are close to the ours (compare columns 10 and 6, Table 3). This proves the correctness of our experimental and calculated results for the intensity characteristics of YAlO$_3$:Er^{3+} laser crystals and eliminates the conflict between our data and those reported in [27]. This conclusion is also confirmed by the fact that the calculated radiative lifetimes $\tau^{\text{calc}}_{\text{rad}}$ for most of the initial luminescence J states of Er^{3+} ions in YAlO$_2$ crystals are in a fairly good agreement with the corresponding experimental data on $\tau^{\text{exp}}_{\text{lum}}$ and the nonradiative multiphonon transition probabilities $W_{JJ'}$ (see Table 4).

In summary, it is clearly seen from Table 3 that elimination of several high-lying J levels of Er^{3+} ions from the intensity analysis results in a dramatic improvement of the agreement between calculated and observed intensities of intermanifold transitions of YAlO$_3$:Er^{3+} crystals (thus, the standard of rmc deviation decreases by a factor of about four and becomes as small as 0.046×10^{-20} cm^2, a value typical of the most favorable fitting intensity calculations). Namely, the $^2P_{3/2}$, $^4G_{7/2}$, $^2D(1)_{5/2}$, $^2G(1)_{9/2}$, $^2D_{5/2}$, and $^4D_{7/2}$ manifolds lying above 30000 cm^{-1} were excluded for which we were not able to derive a correct estimate of the integrated absorption $\int k(\lambda)\,d\lambda$ coefficients because of the presence of some unidentified sources of strong optical absorption.

4. A Full Set of Squared Reduced-Matrix Elements $|\langle 4f^N\alpha[SL]\,J|\,|U^{(t)}|\,|4f^N\alpha'[S'L']\,J'\rangle|^2$ for Intermanifold $J \to J'$ Transitions of Er^{3+} Ions[6]

Because of the electron–hole symmetry, the Er^{3+} ion has an electronic structure quite similar to that of the Nd^{3+} ion. In particular, the lists of terms and J manifolds of Er^{3+} and Nd^{3+} ions are identical and involve 17 ^{2S+1}L terms and 41 $^{2S+1}L_J$ manifolds (J states), respectively. The principal differences in the 4f electronic structure between these ions are

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[6] Calculations of reduced-matrix elements $\langle \| U^{(t)} \| \rangle$ for Er^{3+} ions (as well as for Nd^{3+} ions in our previous paper [29]) were carried out in the Institute of Crystallography, Russian Academy of Science, Moscow, using the computer program FNCF-93 elaborated by the Russian authors of this paper.

the inverse sequence of J manifolds stemming from the same terms and larger energy spacing for the Er^{3+} ion ($\approx 97000\ cm^{-1}$ versus $\approx 67000\ cm^{-1}$ for Nd^{3+}) resulting from the negative sign of the spin–orbit coupling constant, and stronger localization of 4f electrons for the Er^{3+} ion. As a consequence, the energy separation between J states of the Er^{3+} ion is normally larger than that in the Nd^{3+} ion, so Er^{3+} ions doped into various crystalline hosts exhibit comparatively many initial laser states [1, 2].

The matrix elements for Er^{3+} ions were calculated as in our previous paper [29] for Nd^{3+} ions in the intermediate coupling scheme using eigenfunctions of the parametric atomic Hamiltonian whose structure was described elsewhere [37],

$$H = \zeta_{4f} \sum_i l_i s_i + \sum_{k=0,2,4,6} f_k F^k + \alpha L(L+1) + \beta G(G_2) + \gamma G(R_7)\,, \tag{9}$$

where ζ_{4f} is the spin–orbit coupling constant for 4f electrons, f_k and F^k ($k = 0, 2, 4, 6$) are angular and radial parameters of Coulomb interactions between 4f electrons, respectively (i.e. F^k are the radial Slater integrals). These terms represent the most strong interactions forming the J manifold structure of the energy spectrum of $4f^N$ configuration of a free Ln^{3+} ion. In contrast, three last terms in (9) containing the generalized Trees parameters α, β, and γ refer to weaker interactions describing coupling between the basic $4f^N$ configuration and various excited configurations of the corresponding lanthanide ion. In these terms, L is the operator of the total orbital moment, whereas $G(G_2)$ and $G(R_7)$ are the Casimir operators for the G_2 and R_7 groups, respectively. The correctional Trees parameters cause energy shifts of J manifolds of the order of some hundred wavenumbers and they are absolutely necessary to get the correct positions of baricenters of J manifolds.

The atomic Hamiltonian (9) was diagonalized on the full basis of the $4f^{11}$ configuration of the Er^{3+} ion involving 41 RS J manifolds. The eigenfunctions of the Hamiltonian (9) in the intermediate coupling scheme are J manifolds $|4f^{11}\alpha[SL]\,J\rangle$ which can be expressed via linear combinations of the RS wavefunctions, $|4f^{11}\alpha SLJ\rangle$ (in these notations, the square brackets $[SL]$ reflect the fact that the total spin S and the total angular moment L are no longer good quantum numbers in the intermediate coupling scheme of the $4f^{11}$ configuration). Thus, we have

$$|4f^{11}\alpha[SL]\,J\rangle = \sum_{\alpha' S' L'} C(\alpha[SL]\,J;\alpha' S' L')\,|4f^{11}\alpha' S' L' J'\rangle\,, \tag{10}$$

where the sum runs over all RS terms $\alpha' S' L'$ of the $4f^{11}$ configuration of the Er^{3+} ion. Using the expansion coefficients $C(\alpha[SL]\,J;\alpha' S' L')$ in (10), we can calculate the reduced-matrix elements,

$$\begin{aligned}\langle 4f^{11}\alpha[SL]\,J\|\,U^{(t)}\|\,4f^{11}\alpha'[S'L']\,J'\rangle &= \sum_{\alpha_1 L_1 S_1}\ \sum_{\alpha_2 L_2 S_2} C(\alpha[SL]\,J;\alpha_1 L_1 S_1)\\ &\times C(\alpha'[S'L']\,J';\alpha_2 L_2 S_2)\,\delta(S,S')\,(-1)^{S+L'+J+t}\,[(2J+1)(2J'+1)]^{1/2}\\ &\times \begin{Bmatrix} J & J' & t\\ L' & L & S\end{Bmatrix} (f^N\alpha_1 L_1 S_1\|\,U^{(t)}\|\,f^N\alpha_2 L_2 S_2)\,,\end{aligned} \tag{11}$$

where J, L, S and J', L', S' are the total angular momenta, the total orbital momenta, and the total spins of the initial and final states involved in the intermanifold $J \rightarrow J'$ transition,

Table 5

Squared reduced-matrix elements $|\langle 4f^{11}\alpha[SL]J|\,|U^{(t)}|\,|4f^{11}\alpha'[S'L']J'\rangle|^2$ for intermanifold $J \rightarrow J'$ transitions of Er^{3+} ions

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
$^4I_{13/2}$	$^4I_{15/2}$	6500	0.0195	0.1173	1.4316
$^4I_{11/2}$	$^4I_{13/2}$	3600	0.0331	0.1708	1.0864
	$^4I_{15/2}$	10100	0.0282	0.0003	0.3953
$^4I_{9/2}$	$^4I_{11/2}$	2150	0.0030	0.0674	0.1271
	$^4I_{13/2}$	5750	0.0004	0.0106	0.7162
	$^4I_{15/2}$	12250	0	0.1732	0.0099
$^4F_{9/2}$	$^4I_{9/2}$	2850	0.1279	0.0059	0.0281
	$^4I_{11/2}$	5050	0.0704	0.0112	1.2839
	$^4I_{13/2}$	8650	0.0101	0.1533	0.0714
	$^4I_{15/2}$	15150	0	0.5354	0.4619
$^4S_{3/2}$	$^4F_{9/2}$	3200	0	0.0003	0.0264
	$^4I_{9/2}$	6100	0	0.0788	0.2542
	$^4I_{11/2}$	8250	0	0.0042	0.0739
	$^4I_{13/2}$	11850	0	0	0.3462
	$^4I_{15/2}$	18350	0	0	0.2211
$^2H(2)_{11/2}$	$^4S_{3/2}$	800	0	0.1988	0.0101
	$^4F_{9/2}$	4000	0.3629	0.0224	0.0022
	$^4I_{9/2}$	6900	0.2077	0.0662	0.2858
	$^4I_{11/2}$	9050	0.0357	0.1382	0.0371
	$^4I_{13/2}$	12650	0.0230	0.0611	0.0527
	$^4I_{15/2}$	19150	0.7125	0.4123	0.0925
$^4F_{7/2}$	$^2H(2)_{11/2}$	1150	0.1229	0.0153	0.4017
	$^4S_{3/2}$	1950	0.0001	0.0058	0
	$^4F_{9/2}$	5200	0.0121	0.0342	0.0151
	$^4I_{9/2}$	8050	0.0163	0.0954	0.4277
	$^4I_{11/2}$	10200	0.0035	0.2648	0.1515
$^4F_{5/2}$	$^4I_{13/2}$	13800	0	0.3371	0.0001
	$^4I_{15/2}$	20300	0	0.1468	0.6266
	$^4F_{7/2}$	1650	0.0765	0.0503	0.1015
	$^2H(2)_{11/2}$	2800	0	0.0586	0.1825
	$^4S_{3/2}$	3600	0.0082	0.0040	0
	$^4F_{9/2}$	6850	0.0004	0.2415	0.3575
	$^4I_{9/2}$	9700	0.0107	0.0576	0.1020
	$^4I_{11/2}$	11850	0	0.0979	0.0028
	$^4I_{13/2}$	15450	0	0.1783	0.3429
	$^4I_{15/2}$	21950	0	0	0.2233
$^4F_{3/2}$	$^4F_{5/2}$	350	0.0618	0.0350	0
	$^4F_{7/2}$	2000	0.0028	0.0584	0
	$^4H(2)_{11/2}$	3150	0	0.0005	0.0030
	$^4S_{3/2}$	3950	0.0260	0	0
	$^4F_{9/2}$	7200	0	0.0040	0.0595
	$^4I_{9/2}$	10050	0	0.2299	0.0558
	$^4I_{11/2}$	12200	0	0.0927	0.4861
	$^4I_{13/2}$	15800	0	0	0.0345
	$^4I_{15/2}$	22300	0	0	0.1272
$^2H(2)_{9/2}$	$^4F_{3/2}$	2100	0	0.0208	0.0087
	$^4F_{5/2}$	2450	0.0124	0.0259	0.0063
	$^4F_{7/2}$	4100	0.1058	0.0488	0.0240
	$^2H(2)_{11/2}$	5250	0.0308	0.1828	0.0671
	$^4S_{3/2}$	6050	0	0.0019	0.0025
	$^4F_{9/2}$	9250	0.0055	0.0314	0.0369
	$^4I_{9/2}$	12150	0.0147	0.0062	0.0043
	$^4I_{11/2}$	14300	0.0428	0.0824	0.1128
	$^4I_{13/2}$	17900	0.0780	0.1194	0.3535
	$^4I_{15/2}$	24400	0	0.0190	0.2255

Table 5 (continued)

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
$^4G_{11/2}$	$^2H(2)_{9/2}$	2000	0.2906	0.1170	0.1328
	$^4F_{3/2}$	4100	0	0.0234	0.0923
	$^4F_{5/2}$	4450	0	0.0378	0.0815
	$^4F_{7/2}$	6100	0.0877	0.1287	0.0159
	$^2H(2)_{11/2}$	7250	0.0004	0.1539	0.0494
	$^4S_{3/2}$	8050	0	0.1302	0.0044
	$^4F_{9/2}$	11250	0.4252	0.0368	0.0122
	$^4I_{9/2}$	14100	0.0716	0.0131	0.0235
	$^4I_{11/2}$	16300	0.0003	0.0496	0.0134
	$^4I_{13/2}$	19900	0.1013	0.2651	0.2594
	$^4I_{15/2}$	26400	0.9181	0.5261	0.1171
$^4G_{9/2}$	$^4G_{11/2}$	1000	0.0005	0.2021	0.1639
	$^2H(2)_{9/2}$	2950	0.0269	0	0.0452
	$^4F_{3/2}$	5050	0	0.1710	0.1089
	$^4F_{5/2}$	5400	0.1630	0.0824	0.0028
	$^4F_{7/2}$	7050	0.6062	0.0088	0.1243
	$^2H(2)_{11/2}$	8200	0.0218	0.3274	0.1495
	$^4S_{3/2}$	9000	0	0.1651	0.0100
	$^4F_{9/2}$	12250	0.2201	0.3121	0.3765
	$^4I_{9/2}$	15100	0.0051	0.0042	0.0027
	$^4I_{11/2}$	17250	0.0894	0.1524	0.0144
	$^4I_{13/2}$	20850	1.0908	0.3520	0.0160
	$^4I_{15/2}$	27350	0	0.2415	0.1234
$^2K_{15/2}$	$^4G_{9/2}$	300	0	0.0114	0.0598
	$^4G_{11/2}$	1300	0.0965	0.0595	0.6706
	$^2H(2)_{9/2}$	3300	0	0.7106	0.0758
	$^4F_{3/2}$	5400	0	0	0.0001
	$^4F_{5/2}$	5750	0	0	0.0461
	$^4F_{7/2}$	7400	0	0.0001	0.0002
	$^2H(2)_{11/2}$	8550	0.0977	0.0001	1.1458
	$^4S_{3/2}$	9350	0	0	0.0032
	$^4F_{9/2}$	12550	0	0.0776	0.0125
	$^4I_{9/2}$	15400	0	0.2221	0.1003
	$^4I_{11/2}$	17600	0.0468	0.0018	0.2488
	$^4I_{13/2}$	21200	0.0001	0.0016	0.0261
	$^4I_{15/2}$	27700	0.0219	0.0041	0.0757
$^2G(1)_{7/2}$	$^2K_{15/2}$	200	0	0.1154	0.0026
	$^4G_{9/2}$	500	0.0041	0.1891	0.1582
	$^2G_{11/2}$	1500	0.0150	0.0604	0.0193
	$^2H(2)_{9/2}$	3450	0.0145	0.0056	0.0205
	$^4F_{3/2}$	5550	0.0941	0.0314	0
	$^4F_{5/2}$	5900	0.3716	0.0023	0.0378
	$^4F_{7/2}$	7550	0.1239	0.0424	0.0071
	$^2H(2)_{11/2}$	8700	0.0019	0.0344	0.2672
	$^4S_{3/2}$	9500	0.0445	0.1594	0
	$^4F_{9/2}$	12750	0.0003	0.0078	0.0128
	$^4I_{9/2}$	15600	0.1586	0.3607	0.2204
	$^4I_{11/2}$	17750	0.4934	0.2708	0.1674
	$^4I_{13/2}$	21350	0	0.1009	0.0312
	$^4I_{15/2}$	27850	0	0.0174	0.1163
$^2D(1)_{3/2}$	$^2G(1)_{7/2}$	3650	0.0125	0.0004	0
	$^2K_{15/2}$	3850	0	0	0.0268
	$^4G_{9/2}$	4150	0	0.0125	0.0053
	$^4G_{11/2}$	5150	0	0.0266	0.0107
	$^2H(2)_{9/2}$	7150	0	0.2083	0.2591
	$^4F_{3/2}$	9250	0.0123	0	0
	$^4F_{5/2}$	9600	0.0173	0.0433	0
	$^4F_{7/2}$	11250	0.0211	0.0076	0
	$^2H(2)_{11/2}$	12400	0	0.0168	0.0263
	$^4S_{3/2}$	13200	0.0813	0	0

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^{4}F_{9/2}$	16400	0	0.0464	0.0060
	$^{4}I_{9/2}$	19250	0	0.0461	0.0041
	$^{4}I_{11/2}$	21450	0	0.0995	0.0400
	$^{4}I_{13/2}$	25050	0	0	0.1478
	$^{4}I_{15/2}$	31550	0	0	0.0172
$^{2}K_{13/2}$	$^{2}D(1)_{3/2}$	1450	0	0	0.4069
	$^{2}G(1)_{7/2}$	5100	0	0.4801	0.0029
	$^{2}K_{15/2}$	5300	0.1731	0.1134	0.0006
	$^{4}G_{9/2}$	5600	0.0103	0	0.1781
	$^{4}G_{11/2}$	6600	0.0247	0.0084	0.2059
	$^{2}H(2)_{9/2}$	8600	0.0244	0.1829	0.4613
	$^{4}F_{3/2}$	10650	0	0	0.1050
	$^{4}F_{5/2}$	11000	0	0.0006	0.0051
	$^{4}F_{7/2}$	12650	0	0.0559	0.0073
	$^{2}H(2)_{11/2}$	13850	0.0079	0.0051	0.3144
	$^{4}S_{3/2}$	14600	0	0	0.1233
	$^{4}F_{9/2}$	17850	0.0010	0.0163	0.0025
	$^{4}I_{9/2}$	20700	0.0310	0.0744	0.2626
	$^{4}I_{11/2}$	22850	0.0011	0.0017	0.1466
	$^{4}I_{13/2}$	26500	0.0047	0.0018	0.0016
	$^{4}I_{15/2}$	33000	0.0032	0.0029	0.0152
$^{4}G_{5/2}$	$^{2}K_{13/2}$	300	0	0.0121	0.0343
	$^{2}D(1)_{3/2}$	1750	0.1090	0.0004	0
	$^{2}G(1)_{7/2}$	5400	0.0019	0.2228	0.1258
	$^{2}K_{15/2}$	5600	0	0	0.0004
	$^{4}G_{9/2}$	5900	0.0197	0.0669	0.2650
	$^{4}G_{11/2}$	6900	0	0.0005	0.0354
	$^{2}H(2)_{9/2}$	8900	0.0267	0.0348	0.0303
	$^{4}F_{3/2}$	10950	0.3341	0.0838	0
	$^{4}F_{5/2}$	11300	0.3110	0.1203	0
	$^{4}F_{7/2}$	12950	0.0396	0.1260	0.1516
	$^{2}H(2)_{11/2}$	14150	0	0	0.1073
	$^{4}S_{3/2}$	14950	0.0399	0.1301	0
	$^{4}F_{9/2}$	18150	0.2476	0.1875	0.1314
	$^{4}I_{9/2}$	21000	0.5866	0.2136	0.0020
	$^{4}I_{11/2}$	23150	0	0.3365	0.0555
	$^{4}I_{13/2}$	26800	0	0.0274	0.0516
	$^{4}I_{15/2}$	33300	0	0	0.0026
$^{2}P_{1/2}$	$^{4}G_{5/2}$	50	0.0811	0	0
	$^{2}K_{13/2}$	350	0	0	0.0383
	$^{2}D(1)_{3/2}$	1800	0.0047	0	0
	$^{2}G(1)_{7/2}$	5450	0	0.0245	0
	$^{2}K_{15/2}$	5650	0	0	0
	$^{4}G_{9/2}$	5950	0	0.0078	0
	$^{4}G_{11/2}$	6950	0	0	0.0250
	$^{2}H(2)_{9/2}$	8950	0	0	0
	$^{4}F_{3/2}$	11050	0.0361	0	0
	$^{4}F_{5/2}$	11400	0.0078	0	0
	$^{4}F_{7/2}$	13050	0	0.0217	0
	$^{2}H(2)_{11/2}$	14200	0	0	0.1691
	$^{4}S_{3/2}$	15000	0.0057	0	0
	$^{4}F_{9/2}$	18200	0	0.0493	0
	$^{4}I_{9/2}$	21050	0	0.0256	0
	$^{4}I_{11/2}$	23250	0	0	0.0324
	$^{4}I_{13/2}$	26850	0	0	0.0002
	$^{4}I_{15/2}$	33350	0	0	0
$^{4}G_{7/2}$	$^{2}P_{1/2}$	550	0	0.0039	0
	$^{4}G_{5/2}$	650	0.0011	0.1210	0.0375
	$^{2}K_{13/2}$	950	0	0.4962	0.0210
	$^{2}D(1)_{3/2}$	2350	0.0224	0.1150	0

Table 5 (continued)

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^2G(1)_{7/2}$	6050	0.0424	0.0016	0.0396
	$^2K_{15/2}$	6200	0	0.0458	0.0293
	$^4G_{9/2}$	6550	0.0008	0.2468	0.0292
	$^4G_{11/2}$	7500	0.0026	0.0176	0.5057
	$^2H(2)_{9/2}$	9500	0.0978	0.0179	0.4051
	$^4F_{3/2}$	11600	0.0438	0.0001	0
	$^4F_{5/2}$	11950	0.2058	0.0060	0.1324
	$^4F_{7/2}$	13600	0.1418	0.0547	0.0038
	$^2H(2)_{11/2}$	14750	0.0542	0.0019	0.0006
	$^4S_{3/2}$	15550	0.0106	0.1469	0
	$^4F_{9/2}$	18800	0	0.0010	0.0275
	$^4I_{9/2}$	21650	0.0252	0.1155	0.0163
	$^4I_{11/2}$	23800	0.5295	0.0914	0.0523
	$^4I_{13/2}$	27400	0	0.1933	0.0524
	$^4I_{15/2}$	33900	0	0.0334	0.0028
$^2D(1)_{5/2}$	$^4G_{7/2}$	800	0.0005	0.0126	0.0005
	$^2P_{1/2}$	1350	0.0051	0	0
	$^4G_{5/2}$	1400	0.0021	0.0064	0
	$^2K_{13/2}$	1700	0	0.0025	0.0212
	$^2D(1)_{3/2}$	3150	0.0316	0.0062	0
	$^2G(1)_{7/2}$	6800	0.0458	0.0059	0.1986
	$^2K_{15/2}$	7000	0	0	0.3711
	$^4G_{9/2}$	7300	0.2169	0.0001	0.1366
	$^4G_{11/2}$	8300	0	0.0267	0.0051
	$^2H(2)_{9/2}$	10300	0.0264	0.0074	0.0143
	$^4F_{3/2}$	12400	0.0195	0.0119	0
	$^4F_{5/2}$	12750	0.0595	0.0381	0
	$^4F_{7/2}$	14400	0.0148	0.0001	0.0002
	$^2H(2)_{11/2}$	15550	0	0.0778	0.0284
	$^4S_{3/2}$	16350	0.0431	0.0027	0
	$^4F_{9/2}$	19550	0.0361	0.1067	0.0047
	$^4I_{9/2}$	22400	0.0224	0.0026	0.0025
	$^4I_{11/2}$	24600	0	0.0699	0.0636
	$^4I_{13/2}$	28200	0	0.0130	0.0669
	$^4I_{15/2}$	34700	0	0	0.0227
$^2G(1)_{9/2}$	$^2D(1)_{5/2}$	1750	0.0084	0.1185	0.0496
	$^4G_{7/2}$	2550	0.0179	0.0044	0.1769
	$^2P_{1/2}$	3100	0	0.0441	0
	$^4G_{5/2}$	3200	0.0050	0.0289	0.0219
	$^2K_{13/2}$	3500	0.0188	0.0256	0.3429
	$^2D(1)_{3/2}$	4900	0	0.0085	0.0183
	$^2G(1)_{7/2}$	8600	0.0074	0.0256	0.0980
	$^2K_{15/2}$	8750	0	0.4361	0.0861
	$^4G_{9/2}$	9100	0.0024	0.0388	0.0007
	$^4G_{11/2}$	10050	0.0738	0.0014	0.5232
	$^2H(2)_{9/2}$	12050	0.0814	0.0087	0.0259
	$^4F_{3/2}$	14150	0	0.0193	0.0219
	$^4F_{5/2}$	14500	0.0141	0.0070	0.0085
	$^4F_{7/2}$	16150	0.0473	0.0008	0.0485
	$^2H(2)_{11/2}$	17300	0.0172	0.0641	0.2129
	$^4S_{3/2}$	18100	0	0.0234	0.0018
	$^4F_{9/2}$	21350	0.0058	0.0035	0.0008
	$^4I_{9/2}$	24200	0.0044	0.0004	0.0149
	$^4I_{11/2}$	26350	0.0089	0.0212	0.0260
	$^4I_{13/2}$	29950	0.1429	0.0145	0.0178
	$^4I_{15/2}$	36450	0	0.0500	0.0001
$^4D_{5/2}$	$^2G(1)_{9/2}$	2000	0.0015	0.1106	0.0654
	$^2D(1)_{5/2}$	3750	0.1916	0.0625	0
	$^4G_{7/2}$	4550	0.0277	0.0993	0.1373
	$^2P_{1/2}$	5100	0.0894	0	0

Table 5 (continued)

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4G_{5/2}$	5200	0.0166	0.0050	0
	$^2K_{13/2}$	5500	0	0.0464	0.0499
	$^2D(1)_{3/2}$	6900	0.0826	0.0073	0
	$^2G(1)_{7/2}$	10600	0.2192	0.0303	0.0002
	$^2K_{15/2}$	10750	0	0	0.1093
	$^4G_{9/2}$	11100	0.2030	0.0032	0.0324
	$^4G_{11/2}$	12050	0	0.1159	0.0726
	$^2H(2)_{9/2}$	14050	0.1270	0.0042	0.0022
	$^4F_{3/2}$	16150	0.1372	0.1930	0
	$^4F_{5/2}$	16500	0.1753	0.0168	0
	$^4F_{7/2}$	18150	0.0935	0.0833	0.0070
	$^2H(2)_{11/2}$	19300	0	0.0625	0.0314
	$^4S_{3/2}$	20100	0.2122	0.0119	0
	$^4F_{9/2}$	23350	0.2375	0.3637	0.0167
	$^4I_{9/2}$	26200	0.0528	0.0114	0.0380
	$^4I_{11/2}$	28350	0	0.1809	0.0023
	$^4I_{13/2}$	31950	0	0.3439	0.0061
	$^4I_{15/2}$	38450	0	0	0.0267
$^4D_{7/2}$	$^4D_{5/2}$	600	0.1835	0.0892	0.0019
	$^2G(1)_{9/2}$	2600	0.0026	0.0004	0.0077
	$^2D(1)_{5/2}$	4350	0.1418	0.1106	0.0045
	$^4G_{7/2}$	5150	0.0009	0.0006	0.1449
	$^2P_{1/2}$	5700	0	0.0379	0
	$^4G_{5/2}$	5750	0.0015	0.0065	0.2821
	$^2K_{13/2}$	6050	0	0.0001	0.0145
	$^2D(1)_{3/2}$	7500	0.0494	0.0015	0
	$^2G(1)_{7/2}$	11200	0.0156	0.0103	0.1563
	$^2K_{15/2}$	11350	0	0.0019	0.0297
	$^4G_{9/2}$	11700	0.2126	0.0163	0.1740
	$^4G_{11/2}$	12650	0.4818	0.0144	0.0188
	$^2H(2)_{9/2}$	14650	0.1792	0.0120	0.0135
	$^4F_{3/2}$	16750	0.0590	0.0777	0
	$^4F_{5/2}$	17100	0.0429	0.2443	0.0002
	$^4F_{7/2}$	18750	0.2413	0.4226	0.0016
	$^2H(2)_{11/2}$	19900	0.6624	0.0048	0.0355
	$^4S_{3/2}$	20700	0.3378	0.0092	0
	$^4F_{9/2}$	23900	0.3503	0.1443	0.0002
	$^4I_{9/2}$	26800	0.0966	0.0736	0.0030
	$^4I_{11/2}$	28950	0.0119	0.0386	0.0221
	$^4I_{13/2}$	32550	0	0.2643	0.0588
	$^4I_{15/2}$	39050	0	0.8919	0.0291
$^2I_{11/2}$	$^4D_{7/2}$	1850	0.0053	0	0.0161
	$^4D_{5/2}$	2450	0	0.0099	0.0559
	$^2G(1)_{9/2}$	4450	0.0001	0.0331	0.0298
	$^2D(1)_{5/2}$	6200	0	0.0808	0.3178
	$^4G_{7/2}$	7000	0.0066	0.0821	0.0133
	$^2P_{1/2}$	7550	0	0	0.1639
	$^4G_{5/2}$	7600	0	0.0011	0
	$^2K_{13/2}$	7900	0.2556	0.8170	0.4255
	$^2D(1)_{3/2}$	9350	0	0.1999	0.1438
	$^2G(1)_{7/2}$	13050	0.0186	0.0643	0.0562
	$^2K_{15/2}$	13200	0.7852	0.1992	0.0912
	$^4G_{9/2}$	13550	0.0111	0.0274	0.0070
	$^4G_{11/2}$	14500	0.0040	0.2516	0.0089
	$^2H(2)_{9/2}$	16500	0.2757	0.1071	0.2942
	$^4F_{3/2}$	18600	0	0.0695	0.0001
	$^4F_{5/2}$	18950	0	0.0153	0.0464
	$^4F_{7/2}$	20600	0.0061	0.0265	0.0016
	$^2H(2)_{11/2}$	21750	0.0151	0.1361	0.1195
	$^4S_{3/2}$	22550	0	0.0299	0.0688

Table 5 (continued)

$\lvert\alpha[SL]\,J\rangle$	$\lvert\alpha'[S'L']\,J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4F_{9/2}$	25750	0.0014	0.0061	0.0678
	$^4I_{9/2}$	28650	0.0814	0.0821	0.1827
	$^4I_{11/2}$	30800	0.0063	0.0547	0.0272
	$^4I_{13/2}$	34400	0.0024	0.0022	0
	$^4I_{15/2}$	40900	0.0002	0.0285	0.0034
$^2L_{17/2}$	$^2I_{11/2}$	700	0	0.0163	0.1499
	$^4D_{7/2}$	2550	0	0	0.0303
	$^4D_{5/2}$	3100	0	0	0.0623
	$^2G(1)_{9/2}$	5100	0	0	0.1381
	$^2D(1)_{5/2}$	6900	0	0	0.3832
	$^4G_{7/2}$	7650	0	0	0.0274
	$^2P_{1/2}$	8250	0	0	0
	$^4G_{5/2}$	8300	0	0	0.0096
	$^2K_{13/2}$	8600	0.1778	0.4893	0.0569
	$^2D(1)_{3/2}$	10050	0	0	0
	$^2G(1)_{7/2}$	13700	0	0	0.1586
	$^2K_{15/2}$	13900	0.1893	2.0707	0.9750
	$^4G_{9/2}$	14200	0	0.0015	0.0151
	$^4G_{11/2}$	15200	0	0.4876	0.0737
	$^2H(2)_{9/2}$	17200	0	0.0760	0.3134
	$^4F_{3/2}$	19250	0	0	0
	$^4F_{5/2}$	19600	0	0	0.0714
	$^4F_{7/2}$	21250	0	0	0.0011
	$^2H(2)_{11/2}$	22450	0	0.4953	0.3483
	$^4S_{3/2}$	23200	0	0	0
	$^4F_{9/2}$	26450	0	0.0024	0.0664
	$^4I_{9/2}$	29300	0	0.0347	0.1837
	$^4I_{11/2}$	31450	0	0.1819	0.0911
	$^4I_{13/2}$	35100	0.0012	0	0.0036
	$^4I_{15/2}$	41600	0.0047	0.0663	0.0328
$^4D_{3/2}$	$^2L_{17/2}$	550	0	0	0
	$^2I_{11/2}$	1250	0	0.0148	0.0171
	$^4D_{7/2}$	3100	0.1094	0.2233	0
	$^4D_{5/2}$	3700	0.2375	0.0431	0
	$^2G(1)_{9/2}$	5700	0	0.0013	0.0049
	$^2D(1)_{5/2}$	7450	0.0260	0.0025	0
	$^4G_{7/2}$	8250	0.2271	0.0396	0
	$^2P_{1/2}$	8800	0.0032	0	0
	$^4G_{5/2}$	8850	0.0590	0.0006	0
	$^2K_{13/2}$	9150	0	0	0.0518
	$^2D(1)_{3/2}$	10600	0.1305	0	0
	$^2G(1)_{7/2}$	14300	0.1226	0.0149	0
	$^2K_{15/2}$	14450	0	0	0.0470
	$^4G_{9/2}$	14800	0	0.0439	0.1399
	$^4G_{11/2}$	15750	0	0.0031	0.0211
	$^2H(2)_{9/2}$	17750	0	0.1938	0.0404
	$^4F_{3/2}$	19850	0.0114	0	0
	$^4F_{5/2}$	20200	0.0412	0.1037	0
	$^4F_{7/2}$	21850	0.1224	0.0307	0
	$^2H(2)_{11/2}$	23000	0	0.0031	0.1740
	$^4S_{3/2}$	23800	0.1359	0	0
	$^4F_{9/2}$	27000	0	0.0028	0
	$^4I_{9/2}$	29900	0	0.1801	0.0070
	$^4I_{11/2}$	32050	0	0.1624	0.0001
	$^4I_{13/2}$	35650	0	0	0.0282
	$^4I_{15/2}$	42150	0	0	0.0126
$^2P_{3/2}$	$^4D_{3/2}$	700	0.0039	0	0
	$^2L_{17/2}$	1300	0	0	0
	$^2I_{11/2}$	1950	0	0.0788	0.1247
	$^4D_{7/2}$	3800	0.0107	0.0252	0

Table 5 (continued)

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4D_{5/2}$	4400	0.0029	0.0044	0
	$^2G(1)_{9/2}$	6400	0	0.0053	0.1545
	$^2D(1)_{5/2}$	8150	0.1415	0.0194	0
	$^4G_{7/2}$	8950	0.0014	0.0796	0
	$^2P_{1/2}$	9500	0.2056	0	0
	$^4G_{5/2}$	9600	0.0861	0.0020	0
	$^2K_{13/2}$	9900	0	0	0
	$^2D(1)_{3/2}$	11300	0.0019	0	0
	$^2G(1)_{7/2}$	15000	0.1274	0.1162	0
	$^2K_{15/2}$	15150	0	0	0.2355
	$^4G_{9/2}$	15500	0	0.0184	0.0018
	$^4G_{11/2}$	16450	0	0.1226	0.0950
	$^2H(2)_{9/2}$	18450	0	0.0088	0.0003
	$^4F_{3/2}$	20550	0.0137	0	0
	$^4F_{5/2}$	20900	0.0003	0.0972	0
	$^4F_{7/2}$	22550	0.0949	0.0422	0
	$^2H(2)_{11/2}$	23700	0	0.1113	0.0010
	$^4S_{3/2}$	24500	0.0939	0	0
	$^4F_{9/2}$	27700	0	0.0085	0.0042
	$^4I_{9/2}$	30600	0	0.1318	0.0084
	$^4I_{11/2}$	32750	0	0.0192	0.0241
	$^4I_{13/2}$	36350	0	0	0.0161
	$^4I_{15/2}$	42850	0	0	0.0002
$^2I_{13/2}$	$^2P_{3/2}$	750	0	0	0.1740
	$^4D_{3/2}$	1450	0	0	0.0164
	$^2L_{17/2}$	2050	1.2921	0.6922	0.5118
	$^2I_{11/2}$	2700	0.3898	0.3153	0.2094
	$^4D_{7/2}$	4550	0	0.0012	0.0143
	$^4D_{5/2}$	5150	0	0.2035	0.0817
	$^2G(1)_{9/2}$	7150	0.1011	0.0003	0.6238
	$^2D(1)_{5/2}$	8900	0	0.0138	0.1935
	$^4G_{7/2}$	9700	0	0.1187	0.0664
	$^2P_{1/2}$	10250	0	0	0.3384
	$^4G_{5/2}$	10350	0	0.0055	0.0386
	$^2K_{13/2}$	10650	0.0367	0.0430	0.0728
	$^2D(1)_{3/2}$	12050	0	0	0.1761
	$^2G(1)_{7/2}$	15750	0	0.0974	0.0320
	$^2K_{15/2}$	15900	0.1104	0.5552	0.6479
	$^4G_{9/2}$	16250	0.0066	0.0105	0.1048
	$^4G_{11/2}$	17200	0.1257	0.3380	0.0163
	$^2H(2)_{9/2}$	19200	0.0123	0.0608	0.0006
	$^4F_{3/2}$	21300	0	0	0.0095
	$^4F_{5/2}$	21650	0	0.0132	0.0372
	$^4F_{7/2}$	23300	0	0.0082	0.0163
	$^2H(2)_{11/2}$	24450	0.0332	0.1531	0.0964
	$^4S_{3/2}$	25250	0	0	0.1261
	$^4F_{9/2}$	28450	0.0105	0.0081	0.0924
	$^4I_{9/2}$	31350	0.0002	0.0412	0.0019
	$^4I_{11/2}$	33500	0.0192	0.0558	0.0088
	$^4I_{13/2}$	37100	0.0020	0.0027	0.0065
	$^4I_{15/2}$	43600	0.0055	0.0171	0.0050
$^4D_{1/2}$	$^2I_{13/2}$	3300	0	0	0.0286
	$^2P_{3/2}$	4100	0	0	0
	$^4D_{3/2}$	4800	0.2500	0	0
	$^2L_{17/2}$	5350	0	0	0
	$^2I_{11/2}$	6050	0	0	0.0296
	$^4D_{7/2}$	7900	0	0.1708	0
	$^4D_{5/2}$	8450	0.0007	0	0

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^2G(1)_{9/2}$	10450	0	0.0486	0
	$^2D(1)_{5/2}$	12250	0.0768	0	0
	$^4G_{7/2}$	13000	0	0.0159	0
	$^2P_{1/2}$	13600	0	0	0
	$^4G_{5/2}$	13650	0.2827	0	0
	$^2K_{13/2}$	13950	0	0	0.0025
	$^2D(1)_{3/2}$	15400	0.0020	0	0
	$^2G(1)_{7/2}$	19050	0	0.0117	0
	$^2K_{15/2}$	19250	0	0	0
	$^4G_{9/2}$	19550	0	0.0221	0
	$^4G_{11/2}$	20550	0	0	0.1563
	$^2H(2)_{9/2}$	22550	0	0.0095	0
	$^4F_{3/2}$	24600	0.1124	0	0
	$^4F_{5/2}$	24950	0.0835	0	0
	$^4F_{7/2}$	26600	0	0.1760	0
	$^2H(2)_{11/2}$	27800	0	0	0.0456
	$^4S_{3/2}$	28600	0.0298	0	0
	$^4F_{9/2}$	31800	0	0.1627	0
	$^4I_{9/2}$	34650	0	0.0757	0
	$^4I_{11/2}$	36800	0	0	0.0156
	$^4I_{13/2}$	40450	0	0	0.0150
	$^4I_{15/2}$	46950	0	0	0
$^2H(1)_{9/2}$	$^4D_{1/2}$	750	0	0.0178	0
	$^2I_{13/2}$	4100	0.0027	0.0931	0.1594
	$^2P_{3/2}$	4850	0	0.0783	0.0068
	$^4D_{3/2}$	5550	0	0.0626	0.0674
	$^2L_{17/2}$	6150	0	0.0141	0.1926
	$^2I_{11/2}$	6800	0.7491	0.4525	0.1485
	$^4D_{7/2}$	8650	0.0019	0.0005	0.0049
	$^4D_{5/2}$	9250	0.0175	0.1021	0.0077
	$^2G(1)_{9/2}$	11250	0.0081	0.0164	0.0400
	$^2D(1)_{5/2}$	13000	0	0.0720	0.4821
	$^4G_{7/2}$	13800	0.2143	0.0674	0.0222
	$^2P_{1/2}$	14350	0	0.1419	0
	$^4G_{5/2}$	14400	0.0011	0.0113	0.0022
	$^2K_{13/2}$	14750	1.3156	0.0106	0.0480
	$^2D(1)_{3/2}$	16150	0	0.0201	0.0676
	$^2G(1)_{7/2}$	19850	0.1937	0.0370	0.1217
	$^2K_{15/2}$	20000	0	0.0002	0.0636
	$^4G_{9/2}$	20350	0.0019	0.0352	0.0001
	$^4G_{11/2}$	21300	0.0015	0.0759	0.0051
	$^2H(2)_{9/2}$	23300	0.0074	0.2059	0.0189
	$^4F_{3/2}$	25400	0	0.0302	0.0325
	$^4F_{5/2}$	25750	0.0028	0.0230	0.1038
	$^4F_{7/2}$	27400	0.0097	0.0107	0.0011
	$^2H(2)_{11/2}$	28550	0.0041	0.0605	0.0019
	$^4S_{3/2}$	29350	0	0.0149	0.0178
	$^4F_{9/2}$	32550	0	0.0114	0.0214
	$^4I_{9/2}$	35450	0.0018	0.1595	0.0196
	$^4I_{11/2}$	37600	0.0001	0.0654	0.0028
	$^4I_{13/2}$	41200	0.0003	0.0017	0.0001
	$^4I_{15/2}$	47700	0	0.0038	0.0001
$^2L_{15/2}$	$^2H(1)_{9/2}$	100	0	0.1430	1.1625
	$^4D_{1/2}$	900	0	0	0
	$^2I_{13/2}$	4200	0.1213	0.8462	0.0215
	$^2P_{3/2}$	4950	0	0	0.0155
	$^4D_{3/2}$	5650	0	0	0.1360
	$^2L_{17/2}$	6250	0.0614	0.1718	0.2088

Table 5 (continued)

$\lvert\alpha[SL]J\rangle$	$\lvert\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^2I_{11/2}$	6900	0.5330	0.5868	0.4593
	$^4D_{7/2}$	8750	0	0	0.0048
	$^4D_{5/2}$	9350	0	0	0.0169
	$^2G(1)_{9/2}$	11350	0	0.7557	0.4303
	$^2D(1)_{5/2}$	13100	0	0	0.0108
	$^4G_{7/2}$	13900	0	0.0042	0.2496
	$^2P_{1/2}$	14450	0	0	0
	$^4G_{5/2}$	14550	0	0	0.0260
	$^2K_{13/2}$	14850	0.0332	1.4573	1.1610
	$^2D(1)_{3/2}$	16250	0	0	0.0010
	$^2G(1)_{3/2}$	19950	0	0.0141	0.2110
	$^2K_{15/2}$	20100	0.0312	0.0470	0.2539
	$^4G_{9/2}$	20450	0	0.1397	0.1325
	$^4G_{11/2}$	21400	0.0012	0.0158	0.0041
	$^2H(2)_{9/2}$	23400	0	0.0738	0.0016
	$^4F_{3/2}$	25500	0	0	0.0261
	$^4F_{5/2}$	25850	0	0	0.0003
	$^4F_{7/2}$	27500	0	0.0017	0.0511
	$^2H(2)_{11/2}$	28650	0.0008	0.0171	0.0037
	$^4S_{3/2}$	29450	0	0	0.0028
	$^4F_{9/2}$	32650	0	0.0336	0.0486
	$^4I_{9/2}$	35550	0	0.1151	0.0366
	$^4I_{11/2}$	37700	0.0003	0.0018	0.0029
	$^4I_{13/2}$	41300	0.0014	0.0186	0.0104
	$^4I_{15/2}$	47800	0.0002	0.0027	0.0021
$^2D(2)_{5/2}$	$^2L_{15/2}$	1100	0	0	0.0006
	$^2H(1)_{9/2}$	1200	0.0036	0.0588	0.2582
	$^4D_{1/2}$	2000	0.1318	0	0
	$^2I_{13/2}$	5300	0	0.1519	0.0359
	$^2P_{3/2}$	6050	0.1530	0.0023	0
	$^4D_{3/2}$	6750	0.0289	0.0001	0
	$^2L_{17/2}$	7350	0	0	0.4315
	$^2I_{11/2}$	8050	0	0.0111	0.0378
	$^4D_{7/2}$	9900	0.0967	0.0481	0.0036
	$^4D_{5/2}$	10450	0.0097	0.0696	0
	$^2G(1)_{9/2}$	12450	0.1052	0.0237	0.0219
	$^2D(1)_{5/2}$	14250	0.2578	0.0345	0
	$^4G_{7/2}$	15000	0.1343	0.1723	0.0299
	$^2P_{1/2}$	15600	0.0566	0	0
	$^4G_{5/2}$	15650	0.0035	0.0008	0
	$^2K_{13/2}$	15950	0	0.0394	0.0933
	$^2D(1)_{3/2}$	17400	0.1380	0.1481	0
	$^2G(1)_{7/2}$	21050	0.0004	0.1025	0.0259
	$^2K_{15/2}$	21250	0	0	0.0024
	$^4G_{9/2}$	21550	0.3413	0.0022	0.0654
	$^4G_{11/2}$	22550	0	0.1594	0.0714
	$^2H(2)_{9/2}$	24500	0.2488	0.1532	0.0970
	$^4F_{3/2}$	26600	0.0637	0.0117	0
	$^4F_{5/2}$	26950	0.0178	0.0022	0
	$^4F_{7/2}$	28600	0.0897	0.0136	0.0053
	$^2H(2)_{11/2}$	29750	0	0.2553	0.0002
	$^4S_{3/2}$	30550	0.0086	0.0001	0
	$^4F_{9/2}$	33800	0.0006	0.0390	0.0102
	$^4I_{9/2}$	36650	0.0160	0.0007	0.0015
	$^4I_{11/2}$	38800	0	0.0143	0.0061
	$^4I_{13/2}$	42400	0	0.2198	0.0029
	$^4I_{15/2}$	48900	0	0	0.0096
$^2H(1)_{11/2}$	$^2D(2)_{5/2}$	1950	0	0.0176	0.0029
	$^2L_{15/2}$	3100	0.7228	0.1617	0.0408
	$^2H(1)_{9/2}$	3200	0.0618	0.1629	0.4389

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4D_{1/2}$	3950	0	0	0.0008
	$^2I_{13/2}$	7300	0.6957	0.5673	0.3124
	$^2P_{3/2}$	8050	0	0.1224	0.1721
	$^4D_{3/2}$	8750	0	0.0025	0.2470
	$^2L_{17/2}$	9300	0	0.2338	1.02045
	$^2I_{11/2}$	10000	0.0452	0.1120	0.0517
	$^4D_{7/2}$	11850	0.0027	0.0005	0.0018
	$^4D_{5/2}$	12450	0	0.1690	0.0172
	$^2G(1)_{9/2}$	14450	0.3097	0.5866	0.0447
	$^2D(1)_{5/2}$	16200	0	0.1787	0.1190
	$^4G_{7/2}$	17000	0.0366	0	0.1226
	$^2P_{1/2}$	17550	0	0	0.0816
	$^4G_{5/2}$	17600	0	0.0043	0.0223
	$^2K_{13/2}$	17900	0.0260	0.0743	0.3540
	$^2D(1)_{3/2}$	19350	0	0.0331	0.0011
	$^2G(1)_{7/2}$	23000	0.0352	0.0006	0.1477
	$^2K_{15/2}$	23200	0.5322	0.0593	0.1055
	$^4G_{9/2}$	23500	0.0046	0.0291	0.0475
	$^4G_{11/2}$	24500	0.0072	0.0689	0.0205
	$^2H(2)_{9/2}$	26500	0.0366	0.0110	0.0003
	$^4F_{3/2}$	28600	0	0.0016	0.0727
	$^4F_{5/2}$	28950	0	0.0317	0.0012
	$^4F_{7/2}$	30600	0.0004	0.0004	0.0585
	$^2H(2)_{11/2}$	31750	0.0001	0.0548	0.0015
	$^4S_{3/2}$	32550	0	0.0054	0
	$^4F_{9/2}$	35750	0.0124	0.0646	0.0129
	$^4I_{9/2}$	38600	0.0003	0.0085	0.0020
	$^4I_{11/2}$	40800	0.0001	0.0077	0.0049
	$^4I_{13/2}$	44400	0.0003	0.0184	0.0022
	$^4I_{15/2}$	50900	0.0001	0.0083	0
$^2F(2)_{7/2}$	$^2H(1)_{11/2}$	4150	0.0121	0.1375	0.1154
	$^2D(2)_{5/2}$	6150	0.3814	0.0089	0.0241
	$^2L_{15/2}$	7250	0	0.0003	0.0399
	$^2H(1)_{9/2}$	7350	0.0001	0.0335	0.2414
	$^4D_{1/2}$	8100	0	0.0184	0
	$^2I_{13/2}$	11450	0	0.0001	0.1623
	$^2P_{3/2}$	12200	0.0500	0.0663	0
	$^4D_{3/2}$	12900	0.0212	0.0052	0
	$^2L_{17/2}$	13500	0	0	0.4858
	$^2I_{11/2}$	14150	0.0153	0.0094	0.4339
	$^4D_{7/2}$	16000	0.0106	0.0081	0.0021
	$^4D_{5/2}$	16600	0.0766	0	0.0109
	$^2G(1)_{9/2}$	18600	0.0080	0.0218	0.0710
	$^2D(1)_{5/2}$	20350	0.7982	0.0394	0.0101
	$^4G_{7/2}$	21150	0.0105	0.0586	0.0482
	$^2P_{1/2}$	21700	0	0.0942	0
	$^4G_{5/2}$	21750	0.0008	0.0134	0.0118
	$^2K_{13/2}$	22100	0	0.0364	0.4500
	$^2D(1)_{3/2}$	23500	0.1430	0.1269	0
	$^2G(1)_{7/2}$	27200	0.0173	0.0510	0.0517
	$^2K_{15/2}$	27350	0	0.1680	1.0772
	$^4G_{9/2}$	27700	0.0027	0.1191	0.0104
	$^4G_{11/2}$	28650	0.6606	0.4376	0.0004
	$^2H(2)_{9/2}$	30650	0.1869	0.5269	0.0271
	$^4F_{3/2}$	32750	0.0208	0.0034	0
	$^4F_{5/2}$	33100	0.0470	0.0002	0.0027
	$^4F_{7/2}$	34750	0.0001	0.0010	0.0120
	$^2H(2)_{11/2}$	35900	0.4001	0.5224	0.0055
	$^4S_{3/2}$	36700	0.0091	0.0142	0
	$^4F_{9/2}$	39900	0.0002	0.0125	0.0094

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4I_{9/2}$	42800	0.0165	0.1127	0.0003
	$^4I_{11/2}$	44950	0.0530	0.0564	0.0003
	$^4I_{13/2}$	48550	0	0.0012	0.0015
	$^4I_{15/2}$	55050	0	0.0100	0.0003
$^2D(2)_{3/2}$	$^2F(2)_{7/2}$	50	0.0632	0.0010	0
	$^2H(1)_{11/2}$	4200	0	0.0288	0.3238
	$^2D(2)_{5/2}$	6150	0.0361	0.0939	0
	$^2L_{15/2}$	7300	0	0	0.5309
	$^2H(1)_{9/2}$	7400	0	0.0150	0.0316
	$^4D_{1/2}$	8150	0.0696	0	0
	$^2I_{13/2}$	11500	0	0	0.0001
	$^2P_{3/2}$	12250	0.1272	0	0
	$^4D_{3/2}$	12950	0.0015	0	0
	$^2L_{17/2}$	13500	0	0	0
	$^2I_{11/2}$	14200	0	0.0855	0.0042
	$^4D_{7/2}$	16050	0.0071	0.0413	0
	$^4D_{5/2}$	16650	0.0112	0.0287	0
	$^2G(1)_{9/2}$	18650	0	0.0002	0.0026
	$^2D(1)_{5/2}$	20400	0.0003	0.1238	0
	$^4G_{7/2}$	21200	0.3311	0.0186	0
	$^2P_{1/2}$	21750	0.0382	0	0
	$^4G_{5/2}$	21800	0.0151	0	0
	$^2K_{13/2}$	22100	0	0	0.0049
	$^2D(1)_{3/2}$	23550	0.1108	0	0
	$^2G(1)_{7/2}$	27250	0.0374	0.0308	0
	$^2K_{15/2}$	27400	0	0	0.0028
	$^4G_{9/2}$	27750	0	0.0103	0.0403
	$^4G_{11/2}$	28700	0	0.0014	0.1171
	$^2H(2)_{9/2}$	30700	0	0.1702	0.0019
	$^4F_{3/2}$	32800	0.0009	0	0
	$^4F_{5/2}$	33150	0.0188	0.0114	0
	$^4F_{7/2}$	34800	0.0041	0	0
	$^2H(2)_{11/2}$	35950	0	0.0110	0.0148
	$^4S_{3/2}$	36750	0.0032	0	0
	$^4F_{9/2}$	39950	0	0.0003	0.0002
	$^4I_{9/2}$	42850	0	0.0002	0.0005
	$^4I_{11/2}$	45000	0	0.0884	0.0009
	$^4I_{13/2}$	48600	0	0	0.0010
	$^4I_{15/2}$	55100	0	0	0.0008
$^2F(2)_{5/2}$	$^2D(2)_{3/2}$	8050	0.7344	0.0812	0
	$^2F(2)_{7/2}$	8100	0.0700	0.0182	0.0452
	$^2H(1)_{11/2}$	12250	0	0.0283	0.4213
	$^2D(2)_{5/2}$	14200	0.0069	0.0012	0
	$^2L_{15/2}$	15300	0	0	0.4868
	$^2H(1)_{9/2}$	15450	0.0037	0.2173	0.0777
	$^4D_{1/2}$	16200	0.0024	0	0
	$^2I_{13/2}$	19500	0	0.1254	0.2472
	$^2P_{3/2}$	20300	0.1039	0.0672	0
	$^4D_{3/2}$	21000	0.2098	0.0557	0
	$^2L_{17/2}$	21550	0	0	0.0108
	$^2I_{11/2}$	22250	0	0.0598	0.0001
	$^4D_{7/2}$	24100	0.0018	0.0107	0.0001
	$^4D_{5/2}$	24650	0.0030	0.0082	0
	$^2G(1)_{9/2}$	26650	0.6909	0.8962	0.0374
	$^2D(1)_{5/2}$	28450	0.0323	0.0026	0
	$^4G_{7/2}$	29200	0.0057	0.0782	0.0042
	$^2P_{1/2}$	29800	0.0012	0	0
	$^4G_{5/2}$	29850	0.0002	0.0077	0
	$^2K_{13/2}$	30150	0	0.0108	0.3252
	$^2D(1)_{3/2}$	31600	0.0097	0.0246	0

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^2G(1)_{7/2}$	35250	0.0172	0.0056	0.0701
	$^2K_{15/2}$	35450	0	0	0.1110
	$^4G_{9/2}$	35750	0.0525	0.1062	0
	$^4G_{11/2}$	36750	0	0.0126	0.0108
	$^2H(2)_{9/2}$	38750	0.0542	0.0082	0.0287
	$^4F_{3/2}$	40800	0.0145	0.0029	0
	$^4F_{5/2}$	41150	0.0070	0.0002	0
	$^4F_{7/2}$	42800	0.0011	0.0034	0.0002
	$^2H(2)_{11/2}$	44000	0	0.0217	0.0094
	$^4S_{3/2}$	44800	0.0061	0.0056	0
	$^4F_{9/2}$	48000	0.0011	0.0058	0.0022
	$^4I_{9/2}$	50850	0.0207	0.0138	0.0127
	$^4I_{11/2}$	53000	0	0.0093	0.0066
	$^4I_{13/2}$	56650	0	0.0092	0
	$^4I_{15/2}$	63150	0	0	0
$^2G(2)_{7/2}$	$^2F(2)_{5/2}$	2200	0.6037	0.1941	0.0627
	$^2D(2)_{3/2}$	10250	0.0523	0.0028	0
	$^2F(2)_{7/2}$	10300	0.0416	0.0681	0.0018
	$^2H(1)_{11/2}$	14450	0.1520	0.0018	0.0406
	$^2D(2)_{5/2}$	16400	0.1225	0.0199	0.4122
	$^2L_{15/2}$	17500	0	0.7225	0.1904
	$^2H(1)_{9/2}$	17650	1.2412	0.0246	0.0581
	$^4D_{1/2}$	18400	0	0.0161	0
	$^2I_{13/2}$	21700	0	0.0030	0.0184
	$^2P_{3/2}$	22500	0.0837	0.0014	0
	$^4D_{3/2}$	23200	0.0125	0.0073	0
	$^2L_{17/2}$	23750	0	0	0.0345
	$^2I_{11/2}$	24450	1.3825	0.0001	0.0037
	$^4D_{7/2}$	26300	0.0008	0	0.0053
	$^4D_{5/2}$	26850	0.0501	0.0110	0.2296
	$^2G(1)_{9/2}$	28850	0.0297	0.0004	0.0045
	$^2D(1)_{5/2}$	30650	0.0073	0.0749	0.1004
	$^4G_{7/2}$	31400	0.0045	0.1970	0.0092
	$^2P_{1/2}$	32000	0	0.1763	0
	$^4G_{5/2}$	32050	0.0335	0.0050	0.0089
	$^2K_{13/2}$	32350	0	0.0813	0.0719
	$^2D(1)_{3/2}$	33800	0.0530	0.2534	0
	$^2G(1)_{7/2}$	37450	0	0.3000	0.0145
	$^2K_{15/2}$	37650	0	0.1220	0.0230
	$^4G_{9/2}$	37950	0.0020	0.0190	0
	$^4G_{11/2}$	38950	0.0077	0.0231	0.0039
	$^2H(2)_{9/2}$	40950	0.0419	0.2030	0
	$^4F_{3/2}$	43000	0.0170	0.0487	0
	$^4F_{5/2}$	43350	0.0007	0.0237	0.0082
	$^4F_{7/2}$	45000	0.0003	0.0227	0.0010
	$^2H(2)_{11/2}$	46200	0.0003	0.0429	0.0029
	$^4S_{3/2}$	47000	0.0090	0.0868	0
	$^4F_{9/2}$	50200	0.0009	0.0127	0.0001
	$^4I_{9/2}$	53050	0.0135	0.0910	0.0001
	$^4I_{11/2}$	55200	0	0.0200	0.0016
	$^4I_{13/2}$	58850	0	0	0.0003
	$^4I_{15/2}$	65350	0	0.0009	0
$^2G(2)_{9/2}$	$^2G(2)_{7/2}$	4100	0.0880	0.0540	0.1170
	$^2F(2)_{5/2}$	6300	0.1548	0.0404	0.0551
	$^2D(2)_{3/2}$	14350	0	0.0304	0.5583
	$^2F(2)_{7/2}$	14400	0.4808	0.2915	0.0014
	$^2H(1)_{11/2}$	18550	1.4412	0.0059	0.0123
	$^2D(2)_{5/2}$	20500	0.0587	0.0170	0.0613

Table 5 (continued)

$\lvert\alpha[SL]\,J\rangle$	$\lvert\alpha'[S'L']\,J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^2L_{15/2}$	21600	0	0.0512	0.0120
	$^2H(1)_{9/2}$	21750	0.0454	0.0033	0.1153
	$^4D_{1/2}$	22500	0	0.0082	0
	$^2I_{13/2}$	25800	1.5985	0.0004	0.0017
	$^2P_{3/2}$	26600	0	0.0035	0.1919
	$^4D_{3/2}$	27300	0	0.1271	0.0416
	$^2L_{17/2}$	27850	0	1.00038	0.3622
	$^2I_{11/2}$	28550	0.1477	0.0851	0.0933
	$^4D_{7/2}$	30400	0.0057	0.0012	0.0039
	$^4D_{5/2}$	30950	0.1324	0.0839	0.0974
	$^2G(1)_{9/2}$	32950	0.0144	0.4794	0.0002
	$^2D(1)_{5/2}$	34750	0.0360	0.0896	0.0095
	$^4G_{7/2}$	35500	0.0087	0.1305	0.0104
	$^2P_{1/2}$	36100	0	0.0833	0
	$^4G_{5/2}$	36150	0.0011	0.0148	0.0018
	$^2K_{13/2}$	36450	0.0441	0.0084	0.0192
	$^2D(1)_{3/2}$	37900	0	0.0366	0.0169
	$^2G(1)_{7/2}$	41550	0.0019	0.0634	0.0122
	$^2K_{15/2}$	41750	0	0.0535	0.0674
	$^4G_{9/2}$	42050	0.0001	0.0239	0.0002
	$^4G_{11/2}$	43050	0.0688	0.0529	0.0033
	$^2H(2)_{9/2}$	45050	0.0002	0.0899	0.0090
	$^4F_{3/2}$	47100	0	0.0144	0
	$^4F_{5/2}$	47450	0.0077	0.0125	0
	$^4F_{7/2}$	49100	0.0005	0.0259	0.0014
	$^2H(2)_{11/2}$	50300	0.0047	0.0870	0.0010
	$^4S_{3/2}$	51100	0	0.0188	0.0002
	$^4F_{9/2}$	54300	0.0008	0.0673	0.0001
	$^4I_{9/2}$	57150	0	0.0160	0.0027
	$^4I_{11/2}$	59300	0.0026	0.0127	0.0004
	$^4I_{13/2}$	62950	0	0.0030	0.0001
	$^4I_{15/2}$	69450	0	0.0014	0
$^2F(1)_{5/2}$	$^2G(2)_{9/2}$	24000	0.0173	0.1058	0.4246
	$^2G(2)_{7/2}$	28100	1.4271	0.3455	0.0524
	$^2F(2)_{5/2}$	30300	0.3464	0.0086	0
	$^2D(2)_{3/2}$	38350	0.2346	0.0195	0
	$^2F(2)_{7/2}$	38400	0.0007	0.0413	0.5461
	$^2H(1)_{11/2}$	42550	0	0.0605	0.2634
	$^2D(2)_{5/2}$	44500	0	0.1630	0
	$^2L_{15/2}$	45650	0	0	0.7039
	$^2H(1)_{9/2}$	45750	0.9109	0.2792	0.0225
	$^4D_{1/2}$	46500	0.0136	0	0
	$^2I_{13/2}$	49850	0	0.0015	0.1837
	$^2P_{3/2}$	50600	0.0001	0.1895	0
	$^4D_{3/2}$	51300	0.0388	0.0284	0
	$^2L_{17/2}$	51850	0	0	0.0863
	$^2I_{11/2}$	52550	0	0.5573	0.0481
	$^4D_{7/2}$	54400	0.0002	0.0010	0.0089
	$^4D_{5/2}$	55000	0.0003	0.2103	0
	$^2G(1)_{9/2}$	57000	0.0677	0.3147	0.0029
	$^2D(1)_{5/2}$	58750	0.0072	0.0433	0
	$^4G_{7/2}$	59550	0.0780	0.0982	0
	$^2P_{1/2}$	60100	0.1245	0	0
	$^4G_{5/2}$	60150	0.0026	0.0158	0
	$^2K_{13/2}$	60450	0	0.7123	0.0145
	$^2D(1)_{3/2}$	61900	0.0156	0.0119	0
	$^2G(1)_{7/2}$	65550	0.0616	0.1687	0.0241
	$^2K_{15/2}$	65750	0	0	0.0012
	$^4G_{9/2}$	66100	0.0198	0.0256	0.0004

Table 5 (continued)

$\|\alpha[SL]J\rangle$	$\|\alpha'[S'L']J'\rangle$	$E_{JJ'}$ (cm^{-1})	$t = 2$	$t = 4$	$t = 6$
	$^4G_{11/2}$	67050	0	0.0166	0.0127
	$^2H(2)_{9/2}$	69050	0.0002	0	0.0240
	$^4F_{3/2}$	71150	0.0002	0.0126	0
	$^4F_{5/2}$	71500	0	0.0086	0
	$^4F_{7/2}$	73150	0.0069	0.0115	0.0007
	$^2H(2)_{11/2}$	74300	0	0.0033	0.0132
	$^4S_{3/2}$	75100	0.0025	0.0008	0
	$^4F_{9/2}$	78300	0.0045	0.0110	0.0010
	$^4I_{9/2}$	81150	0.0030	0.0021	0.0097
	$^4I_{11/2}$	83350	0	0.0004	0.0057
	$^4I_{13/2}$	86950	0	0.0009	0.0003
	$^4I_{15/2}$	93450	0	0	0
$^2F(1)_{7/2}$	$^2F(1)_{5/2}$	3750	0.1477	0.1453	0.0417
	$^2G(2)_{9/2}$	27800	2.0609	0.6227	0.1828
	$^2G(2)_{7/2}$	31900	0.0558	0.1783	0.4610
	$^2F(2)_{5/2}$	34100	0.0008	0.1412	0.4875
	$^2D(2)_{3/2}$	42100	0.0077	0.0788	0
	$^2F(2)_{7/2}$	42150	0.2469	0.0410	0.1111
	$^2H(1)_{11/2}$	46350	0.7848	0.4953	0.0013
	$^2D(2)_{5/2}$	48300	0.1190	0.0960	0.0241
	$^2L_{15/2}$	49400	0	0.1036	0.0641
	$^2H(1)_{9/2}$	49500	0.0158	0.0153	0.1070
	$^4D_{1/2}$	50300	0	0.0058	0
	$^2I_{13/2}$	53600	0	0.7910	0.1700
	$^2P_{3/2}$	54350	0.1446	0.1445	0
	$^4D_{3/2}$	55050	0.0115	0.0184	0
	$^2L_{17/2}$	55650	0	0	0.6785
	$^2I_{11/2}$	56300	0.1630	0.0113	0.1796
	$^4D_{7/2}$	58150	0.0017	0.0044	0.0031
	$^4D_{5/2}$	58750	0.0232	0.2316	0.0001
	$^2G(1)_{9/2}$	60750	0.0680	0.0433	0.0421
	$^2D(1)_{5/2}$	62500	0.0397	0.0500	0.0005
	$^4G_{7/2}$	63300	0.0097	0.0769	0.0040
	$^2P_{1/2}$	63850	0	0.0222	0
	$^4G_{5/2}$	63950	0.0015	0.0023	0.0107
	$^2K_{13/2}$	64250	0	0.0002	0.0646
	$^2D(1)_{3/2}$	65650	0.0548	0.0781	0
	$^2G(1)_{7/2}$	69350	0.0002	0.0713	0.0040
	$^2K_{15/2}$	69550	0	0.6030	0.0342
	$^4G_{9/2}$	69850	0.0009	0.0001	0.0104
	$^4G_{11/2}$	70850	0.0051	0.0796	0.0124
	$^2H(2)_{9/2}$	72800	0.0388	0.1442	0.0003
	$^4F_{3/2}$	74900	0.0006	0.0228	0
	$^4F_{5/2}$	75250	0.0035	0.0112	0.0015
	$^4F_{7/2}$	76900	0.0022	0.0071	0.0031
	$^2H(2)_{11/2}$	78050	0.0433	0.0300	0.0101
	$^4S_{3/2}$	78850	0.0187	0.0126	0
	$^4F_{9/2}$	82100	0.0116	0.0154	0.0026
	$^4I_{9/2}$	84950	0.0147	0.0375	0.0010
	$^4I_{11/2}$	87100	0.0083	0.0113	0.0013
	$^4I_{13/2}$	90700	0	0.0007	0.0002
	$^4I_{15/2}$	97200	0	0.0056	0.0001

respectively, $\begin{Bmatrix} \cdots \\ \cdots \end{Bmatrix}$ is the $6j$-symbol, and $\delta(\ldots)$ is the Kronecker delta. The values $(f^N\alpha_1L_1S_1|\,|U^{(t)}|\,|f^N\alpha_2L_2S_2)$ occurring in (11) were tabulated in [38] for all RS terms of f^N configurations. In these calculations, we have used the free-ion parameters (ζ_{4f}, F^k, α, β, and γ) reported for Er^{3+} aquo-ions [35]. The matrix elements resulted from our calculations are collected in Table 5.

It should be pointed out that the formal $|4f^{11}\alpha[SL]\,J\rangle$ RS notations for some of the J manifolds in the intermediate coupling scheme may be different for the same Ln^{3+} ion doped into different crystalline hosts (oxides, fluorides, chlorides, etc.). This is related to the fact that the free-ion parameters vary from host to host leading to changes in the expansion coefficients of the principle components in the expansion series of J manifolds over RS manifolds in the intermediate coupling scheme. This can also lead to changes in the sequence of J manifolds in the energy spectrum of a Ln^{3+} ion. If this is the case, one should bear in mind hat different notations can refer to a J manifold which can be identified from the comparison between the corresponding J values, energies $E_{JJ'}$ and the relevant reduced-matrix elements.

5. Conclusion

New spectroscopic and laser data on orthorhombic aluminate $YAlO_3:Er^{3+}$ crystals were obtained at ≈ 110 K under Xe-flashlamp pumping. Green SE was excited in the $^4S_{3/2} \rightarrow {}^4I_{15/2}$ channel for crystals containing 0.5 at% of Er^{3+} activator ions. Cascade laser action at the sequential intermanifold $^4S_{3/2} \rightarrow {}^4I_{11/2} \rightarrow {}^4I_{13/2}$ transitions was obtained for the first time under the same conditions for crystals with enhanced concentration of Er^{3+} ions ($C_{Er} \approx$ 1.5 at%). Quantitative analysis of intensity absorption characteristics of $YAlO_3:Er^{3+}$ crystals was carried out in the frame of the known method [24, 25] and intermanifold radiative transition probabilities and luminescence branching ratios, as well as lifetimes of a number of initial laser states of the Er^{3+} ion were determined. We have revised in detail the intensity spectroscopic parameters Ω_t reported earlier in [27] for $YAlO_3:Er^{3+}$ crystals. In the continuation of our previous paper [29], a full set of reduced-matrix elements $\langle\|U^{(t)}\|\rangle$ for Er^{3+} ions in crystals was calculated to provide a theoretical background for intensity analysis of optical processes involving high-lying states of the activator ion. These data were obtained for the first time and they are thought to be very helpful in numerous practical applications concerning with evaluation of laser potentialities of Er^{3+}-doped insulating crystals, especially those generating SE from upper laser J states under upconversion pumping.

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